INTERACTION OF AN ELASTIC WAVE WITH A PLATE

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UDC 539.41

Within the framework of the dynamic theory of elasticity a numerical solution is set up for an axisymmetric problem which arises in connection with the problem of measuring stresses on the boundary between a solid medium and a rigid wall. In the cylindrical r, z coordinates the medium fills the cylinder $z > 0$, $r < R$, the case $R \to \infty$ being possible when the medium occupies the half-space $z > 0$. The elastic medium borders on a rigid wall which in the plane $z = 0$ has a deformable part in the form of a circular elastic plate clamped along the edges. A plane longitudinal wave in the form of a semiinfinite step falls from infinity. The interaction of this wave with the plate is investigated, with the main attention given to the study of the effect of the problem parameters on the deflection of the plate subjected to the wave.

1. So-called membrane transducers are used to measure stresses at the boundary between a solid medium such as, for example, ground and a rigid wall.

Such a transducer constitutes a metal cylinder with an elastic plate built into one of its ends. It is assumed that the deflection of the center of the plate is proportional to the stress acting on its surface. The coefficient of proportionality is found by placing the device in a liquid and measuring the deflection under the action of a hydrostatic pressure. The body of the device is placed in a wall, so that only the plate is located in the plane of contact.

With this method of measurement, systematic errors arise. They are caused by the fact that the medium possesses a load-carrying capacity, and for the same stresses the deflection of the plate in a liquid will be larger than in a solid. In addition, if the stress within the medium varies sufficiently rapidly, then the plate simply is not able to deflect. It is understood that there may be other errors, caused, for example, by the imperfect apparatus, but they are not considered here.

The errors in question essentially depend on the properties of the medium and the character of variation of the load. In particular, in a liquid at rest they disappear. In the case of real measurements the properties of the medium are known very approximately, especially if the objective of the measurements is to obtain information about these properties. Therefore in the following the medium is assumed to be ideally elastic. We have reasons to believe that for an elastic-plastic medium the static error will be smaller than for an elastic medium, and this is confirmed in a particular case in the paper [1] by this author. If this is so, then the results obtained for an elastic medium can be used also to estimate the errors of measurement in elastic-plastic media.

The inertia properties of the plate most strongly manifest themselves in the case of a sudden variation of the load. Therefore we consider the interaction of the plate with a wave in the form of a semiinfinite step. This allows us to establish the dynamic effects caused by a sudden variation of the load and to obtain the static solution simply by carrying out the calculation as far as reaching the static regime.

The analysis is conducted in cylindrical coordinates. The following notation is used: t is the time, r is the radial coordinate, z is the axial coordinate; u, w are the displacements along r and z; σ_{zz}, σ_{rr}, $\tau \equiv \sigma_{rz}$ are components of the stress tensor, and μ is the shear modulus in the medium. We use dimension-

*Deceased.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 74-85, March-April, 1972. Original article submitted July 23, 1971.

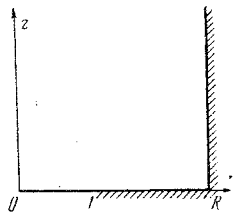

Fig. 1

less variables, chosen so that the density of the medium, the velocity of propagation of the longitudinal wave in the medium, and the radius of the plate equal unity. The connection between the dimensional and dimensionless quantities is described by the expressions

$$x_i = x_i' / L, \quad u_i = u_i' / L, \quad \sigma_{ij} = \sigma_{ij}' / (\lambda' + 2\mu')$$
$$\mu = \mu' / (\lambda' + 2\mu'), \quad t = c't' / L \quad (i = 1, 2) \tag{1.1}$$

where primes are used to denote dimensional variables, $x_1 = r$, $x_2 = z$, $u_1 = u$, $u_2 = w$, σ_{ij} are the components of the stress tensor, λ' and μ' are the Lamé coefficients, L is the dimensional radius of the plate, $c' = \sqrt{(\lambda' + 2\mu')/\rho'}$ is the velocity of propagation of a longitudinal wave, and ρ' is the density of the medium.

The problem is assumed to be axisymmetric. The equations of the theory of elasticity in this case have the form [2]

$$u_{tt} = \mu u_{zz} + u_{rr} + (1 - \mu) w_{rz} + u_r / r - u / r^2$$
$$w_{tt} = w_{zz} + \mu w_{rr} + \mu w_r / r + (1 - \mu)(u_{zr} + u_z / r) \tag{1.2}$$

For these equations we set up the initial and boundary conditions, and they are solved in the region $z > 0$, $0 < r < R$ (Fig. 1).

The initial conditions describe a plane longitudinal wave falling onto the plane $z = 0$ from $z = \infty$; behind the wavefront the medium is in a state of uniaxial strain with $\sigma_{zz} = 1/2$. They have the form

$$u = u_t = 0, \quad w = z / 2, \quad w_t = 1/2 \quad \text{for} \quad z > 0,$$
$$r < R, \quad t = 0 \tag{1.3}$$

The boundary conditions are specified for $r = 0$, $r = R$, and $z = 0$.

The conditions for $r = 0$ are chosen from the symmetry considerations and reduce to the fact that in view of axial symmetry w must be an even and u an odd function of r:

$$u = 0, \quad w_r = 0, \quad \text{for} \quad z > 0, \quad r = 0 \tag{1.4}$$

At $r = R$ the medium borders on a rigid smooth wall on which $u = 0$, $\tau = 0$. It is easy to show that these conditions induce $w_r = 0$ on the wall, and consequently, for $r = R$ we have the same conditions as for $r = 0$, i.e., (1.4).

In the most important case $R \to \infty$, the medium fills the half-space $z > 0$, and the boundary condition for $r = R$ vanishes.

At $z = 0$ two boundary conditions are present.

The first of them corresponds to the assumption about the character of adhesion of the medium to the wall and the plate. Two extreme cases are considered: 1) friction on the boundary is absent (the condition of slipping), 2) $u = 0$ on the boundary (the condition of sticking). In order to write these conditions in a unified form, we introduce the parameter k, which can assume only two values: 0 or 1. The first condition assumes the form

$$ku + (1 - k)\tau = 0 \quad \text{for} \quad z = 0 \tag{1.5}$$

For $k = 0$ it is transformed into the slipping condition, for $k = 1$ it is transformed into the sticking condition, and other values of k are not considered.

The second condition for $z = 0$ arises from the requirement of continuity of the normal displacements on the boundary. Since for $1 \le r \le R$ the medium borders with the rigid wall, then $w = 0$ for these r. For $r < 1$ the medium borders with the elastic plate, and for w the vibration equation of the plate must be satisfied. This equation under the conditions of cylindrical symmetry has the form

$$\rho w_{tt} + I \Delta\Delta w - \sigma_{zz} = 0 \quad \text{for} \quad z = 0, \quad r < 1 \quad (\Delta w = w_{rr} + w_r / r) \tag{1.6}$$

ρ and I are the dimensionless density and rigidity of the plate. In terms of dimensional variables

$$\rho = \frac{\rho_1 d}{\rho' L}, \qquad I = \frac{Ed^3}{12(1-\nu^2)(\lambda' + 2\mu')L^3}$$

where ρ_1 is the density, d is the thickness, L is the radius, E is Young's modulus, and ν is Poisson's ratio of the plate.

For Eq. (1.6), in turn, we have initial and boundary conditions. The initial conditions have the form

$$w = w_t = 0 \quad \text{for} \quad t = 0, \quad z = 0, \; r < 1$$

The boundary condition for $r = 1$ follows from the assumption that the plate is clamped along the edges: $w = w_r = 0$. The condition at $r = 0$ reduces to the fact that in an axisymmetric problem w is an even function of r, i.e., $w' = w''' = 0$ for $r = 0$.

2. The problem thus formulated is solved by the finite-difference method. As usual, the region where the solution is sought is divided by straight lines, parallel to the coordinate axes, into squares with side h. All functions are considered only at the nodes of the resulting grid, at discrete time instants. The derivatives with respect to time and coordinates are replaced by finite-difference relationships. As a result we obtain a system of linear algebraic equations which is solved on a digital computer.

A direct realization of this method is impossible even for the fact that as a result we obtain an infinite system of equations. Therefore the original problem is first transformed: the singularity in the initial conditions is isolated, the infinite region is replaced by a finite region, and Eq. (1.6) is transformed into a more convenient form.

The singularity in the initial conditions (the discontinuity of w_t at $z = 0$) is isolated by representing the initial conditions in the form of a superposition of the static load

$$w = z, \qquad w_t = u_t = u = 0 \qquad \text{for} \quad t = 0 \tag{2.1}$$

and the departing wave

$$w = -z/2, \qquad w_t = {}^1\!/_2, \qquad u = u_t = 0 \qquad \text{for} \quad t = 0 \tag{2.2}$$

In view of linearity of the problem the solution is equal to the sum of the solutions with the initial conditions (2.1) and (2.2). The solution satisfying (2.2) has the form $u = 0$, $w = (t-z)/2$ for $z > t$, and $u = 0$, $w = 0$ for $z \le t$. Consequently, for $z \le t$ the solution with the initial conditions (2.1) coincides with the solution of the original problem.

We are basically interested in w for $t > 0$, $z = 0$; therefore in the following the initial conditions (1.3) are replaced by (2.1), and we do not distinguish between the solution of the original problem and the solution with the initial conditions (2.1).

The subsequent change in the original formulation consists of restricting the region in which the solution is sought. It may be noted that the constant solution $u = 0$, $w = z$ satisfies all conditions of the problem, except the vibration equation of the plate. Since the disturbance caused by the motion of the plate for finite t propagates over a finite region, it is thought natural to seek the solution only in this region. However, this method had to be abandoned, since an increase in the perturbed region imposes inconvenient restrictions on the time up to which the calculation on a BÉSM-3M computer may continue.

In order to avoid these restrictions we introduced a fictitious boundary Γ and assumed that on this boundary the motion was close to a plane one-dimensional motion. For a finite R the boundary Γ was given by a segment of a straight line $z = z_0$, $0 \le r \le R$; for an infinite R the boundary was given by the segments $z = z_0$, $0 \le r \le R_0$ and $r = R_0$, $0 \le z \le z_0$. At the same time z_0 and R_0 were chosen so that the new boundary passed through the nodes of the grid.

We introduce the following notation: $c_1 = 1$, $c_2 = \sqrt{\mu}$, l is the normal to Γ, u^1 and u^2 are the components of the perturbed motion normal and tangent to Γ. In the region $z = z_0$ the quantities thus introduced are given by the equations

$$l = z, \; u^1 = w - z, \; u^2 = u$$

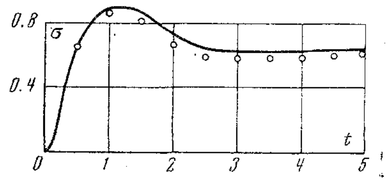

Fig. 2

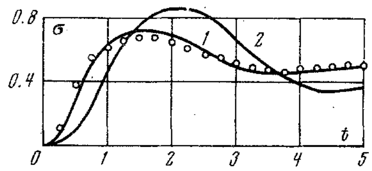

Fig. 3

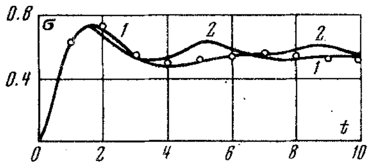

Fig. 4

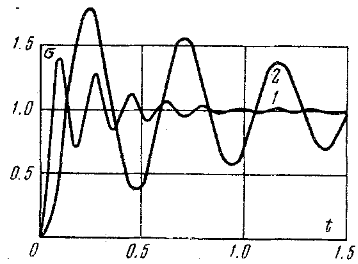

Fig. 5

In the region $r = R_0$ they are

$$l = r,\ u^1 = u,\ u^2 = w - z$$

In terms of these quantities the assumption about the one-dimensionality of the perturbed motion can be written in the form

$$u_{tt}{}^{\alpha} = c_{\alpha}{}^2 u_{ll}{}^{\alpha} \text{ on } \Gamma \ (\alpha = 1, 2) \tag{2.3}$$

The boundary conditions on Γ are taken in the form

$$u_t{}^{\alpha} + c_{\alpha} u_l{}^{\alpha} = 0 \text{ on } \Gamma \ (\alpha = 1, 2) \tag{2.4}$$

In (2.3) and (2.4) there is no summation with respect to α.

It can be shown that in the one-dimensional case the solution of Eqs. (2.3) with the boundary conditions (2.4) coincides with the solution of these equations in the nonbounded region. Use of the conditions (2.4) in a non-one-dimensional problem introduces an error into the solution. The effect of this error is evaluated in Section 4.

Finally, Eq. (1.6) is transformed so that the derivatives with respect to z entering it are computed at the interior points of the region. Denoting by the index h the quantities at $z = h$ and using the equations of motion, we obtain

$$\sigma_{zz} = \sigma_{zz}{}^h - h\partial\sigma_{zz}/\partial z + O(h^2) = \sigma_{zz}{}^h - hw_{tt} + h(\tau_r + \tau / r) + O(h^2) \tag{2.5}$$

From (1.5) we can see that $\tau = k\tau$ for $z = 0$. This unusual equation follows from the fact that k assumes only two values: 0 and 1, with $\tau = 0$ for $k = 0$. Hence we obtain the expression for τ in terms of displacements: $\tau = k\mu w_r + k\mu u_z$. Since on the plate $u = 0$ for $k = 1$, then $ku_z = ku_z^h/h + O(h)$ and

$$h(\tau_r + \tau / r) = k\mu h\Delta w + k\mu h(u_r{}^h + u^h / r) + O(h^2) \tag{2.6}$$

Substituting (2.6) and (2.5) into (1.6) and using the expression for σ_{zz}^h in terms of displacements, we obtain

$$(\rho + h) w_{tt} + I\Delta\Delta w - k\mu h\Delta w = w_z{}^h + (1 - 2\mu + k\mu)(u_r{}^h + u^h / r) + O(h^2) \tag{2.7}$$

3. A setup of the second order of accuracy was chosen for the solution of the problem. The values of u and w at the time instant $t + t_0$ were calculated first at the interior points of the region and then on the boundary with the use of the values already computed.

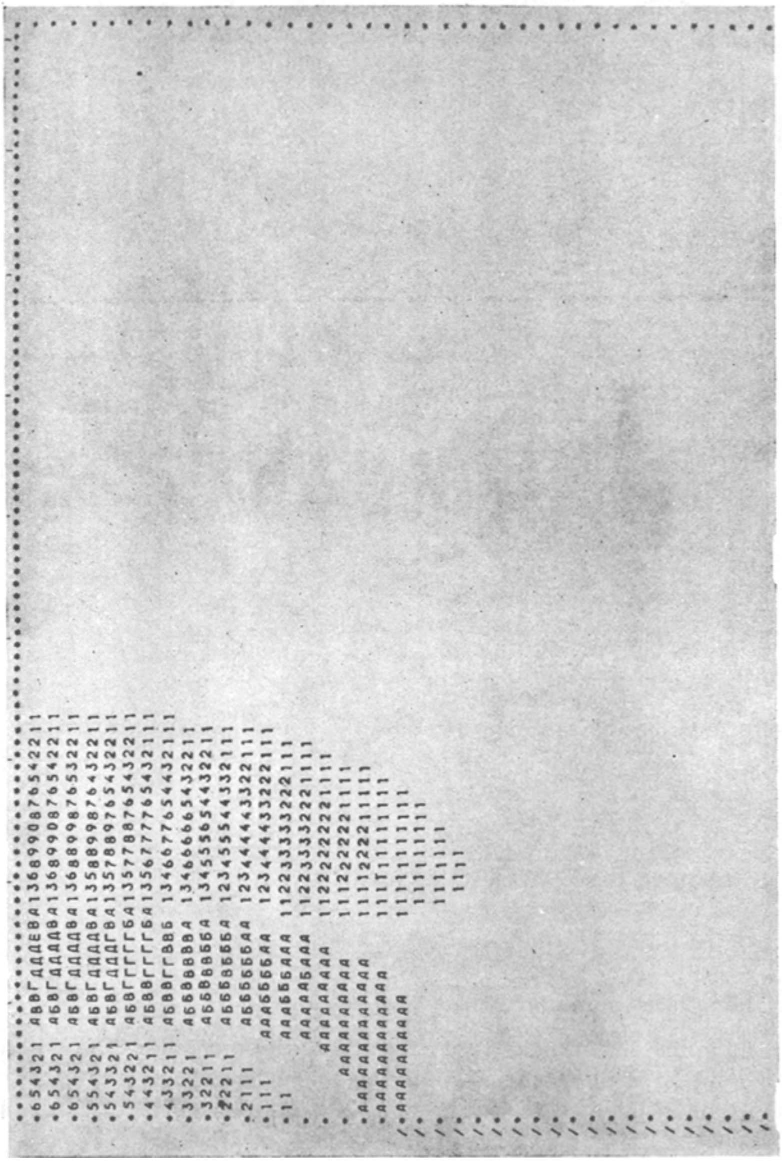

Fig. 6

At the interior points the computations were carried out according to a three-layered explicit scheme for Eqs. (1.2). Here we used the values of u and w already computed at the time instants t and $t-t_0$ within the entire region except the boundary.

The computations on the boundary section $z = 0$, $0 \leq r \leq 1$ were carried out according to a four-layered implicit scheme for Eq. (2.7). The choice of an explicit scheme for Eqs. (1.2) and an implicit scheme for (2.7) is explained by the fact that the computer time basically is expended on the computation of the interior points. Therefore they should be computed by the simplest method, and a scheme which does not impose additional constraints on the step in time should be chosen for the boundary points.

The Courant criterion must be satisfied for stability of the explicit scheme. Therefore we put $t_0 = h/2$, with h chosen so that $1/h$ was an integer.

The additional notation is introduced for the description of the difference scheme:

$$n_1 = 1/h, \ n_2 = z_0/h, \ n_3 = R_1/h, \ R_1 = R$$

if R is finite; $R_1 = R_0$ if R is infinite; l is a normal to the boundary.

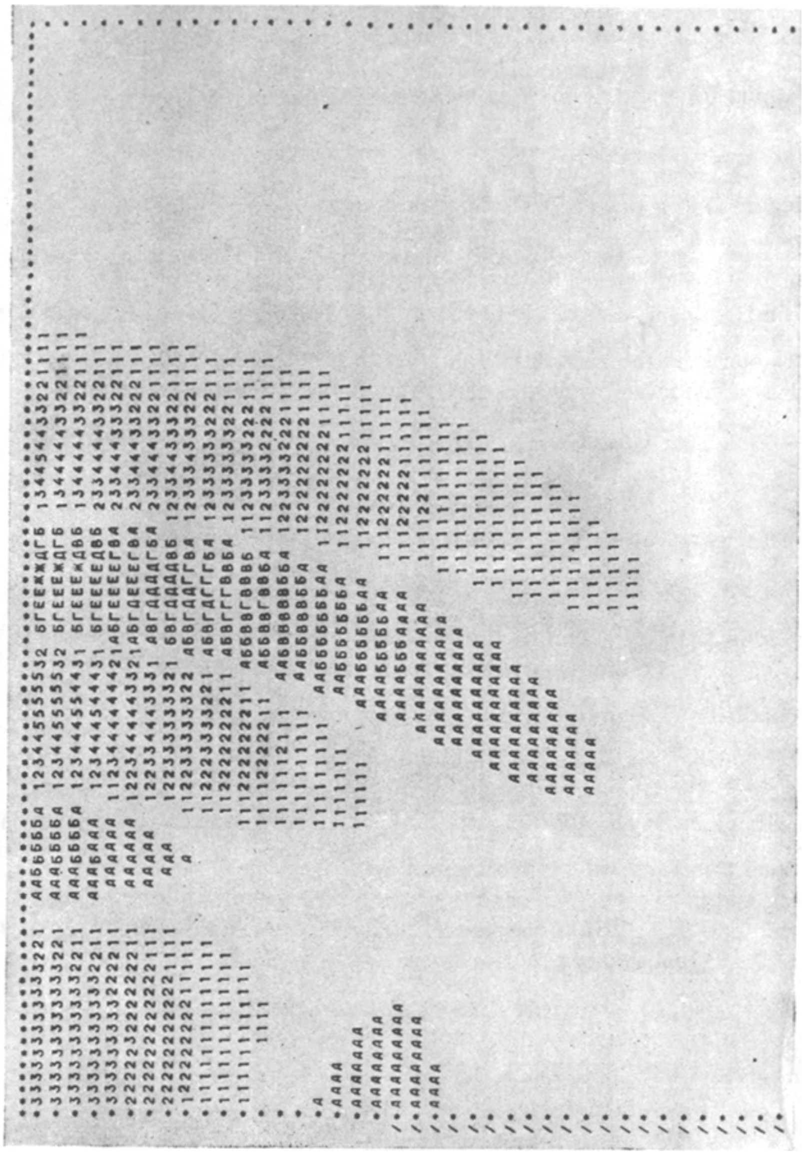

Fig. 7

Derivatives with respect to the coordinates and time are approximated by central differences wherever possible; in the remaining cases one-sided operators are used.

The central and one-sided difference operators are given by the relationships

$$\delta_x f = (f_+ - f_-)\,/\,(2\Delta x), \qquad \delta_{xx} f = (f_+ - 2f + f_-)\,/\,(\Delta x)^2 \tag{3.1}$$

$$\begin{gathered}\delta_{xx}{}^1 f = (3f - 4f_- + f_2)\,/\,(2\Delta x), \qquad \delta_{xx}{}^1 f = (2f - 5f_- + 4f_2 - f_3)\,/\,(\Delta x)^2 \\ f_+ = f\,(x + \Delta x), \quad f_- = f\,(x - \Delta x), \quad f_2 = f\,(x - 2\Delta x), \quad f_3 = f\,(x - 3\Delta x)\end{gathered} \tag{3.2}$$

Here f is an arbitrary function of the argument x, the quantity $\Delta x = t_0$, if x signifies t; $\Delta x = \pm h$ in other cases. The sign of Δx is indifferent for central operators; for one-sided operators it is chosen so that (3.2) does not contain exterior points.

We see that (3.1) and (3.2) approximate the corresponding differential operators with accuracy up to h^2.

If we introduce the abbreviated notation

$$\Delta_1 = \delta_r + r^{-1}, \quad \Delta_r = \Delta_1\delta_r, \quad \Delta_z = \Delta_1\delta_z, \quad \Delta_2 = \Delta_r\Delta_r, \quad \delta_{rz} = \delta_r\delta_z$$

then the difference analog of Eqs. (1.2) can be written in the form

$$\begin{gathered}\delta_{tt} u = \Delta_r u + \mu \delta_{zz} u + (1-\mu)\, \delta_{rz} w - u / r^2 \\ \delta_{tt} w = \delta_{zz} w + \mu \Delta_r w + (1-\mu)\, \Delta_z u\end{gathered} \tag{3.3}$$

where u and w are defined for the argument values

$$t = mt_0, \quad r = ih, \quad z = jh \qquad (m \geqslant 0,\ 0 < i < n_3,\ 0 < j < n_2)$$

with m, i, j being integers. For Eq. (2.7) the approximation

$$(\rho + h)\, \delta_{tt}^1 w + I \Delta_2 w - k\mu h \Delta_r w = \delta_z w + (1 - 2\mu + k\mu)\, \Delta_1 u \tag{3.4}$$

was chosen. Here $t = mt_0$, $r = ih$, $m \geq 1$, $0 \leq i < n_1$; $z = h$ in the right side; $z = 0$ in the left side.

The operator Δ_r contains the singularity $ur = 1$ for $r = 0$, and $\Delta_2 = \Delta_r \Delta_r$ for $r = 0$, $r = h$. At these points the operators Δ_r and Δ_2 were taken, with the use of the boundary conditions for (1.6), in the form

$$\begin{aligned}
&\Delta_r w = 2\, (w_1 - w_0)/h^2 \quad \text{for } r = 0 \\
&\Delta_r w = 2\, (w_0 - w)/h^2 \quad \text{for } r = 1 \\
&\Delta_2 w = 16\, (w_0 - {}^4/_3\, w_1 + w_2/3)/h^4 \text{ for } r = 0 \\
&\Delta_2 w = (-4w + 26 w_0/3 - 20\, w_1/3 + 2w_2)/h^4 \quad \text{for } r = h \\
&w_i = w\,(t + t_0,\ r + ih, 0), \qquad w_- = w\,(t + t_0, r - h, 0)
\end{aligned} \tag{3.5}$$

Equations (3.4) constitute a system of linear equations for the determination of w on the section of the boundary $z = 0$, $0 \leq r < 1$. This system was solved by the run-through method [3].

The boundary condition (1.5) was taken in the form

$$ku + \mu\,(1 - k)\,(\delta_r w + \delta_z^1 u) = 0 \quad \text{for} \quad z = 0, \quad 0 < r \leqslant R_1 \tag{3.6}$$

This condition allows us to calculate u if w is known on the entire lower boundary.

The boundary condition (2.4) was approximated with the use of the central difference relationships. If in (2.4) instead of u_t^α and u_l^α we put $\delta_t u^\alpha$ and $\delta_l u^\alpha$, then the resulting system will contain points lying outside the region. To exclude these points we used a difference approximation of Eqs. (2.3). As a result the boundary condition on Γ was obtained in the form

$$2c_\alpha\,(\delta_t u^\alpha + c_\alpha \delta_l u^\alpha) + h\,(\delta_{tt} u^\alpha - c_\alpha^2 \delta_{ll} u^\alpha) = 0 \tag{3.7}$$

where $\alpha = 1, 2$ and summation with respect to α is absent.

The boundary condition $w_r = 0$ was replaced by $\delta_r^1 w = 0$, while the conditions $u = 0$ and $w = 0$ remained unchanged.

The initial conditions were chosen at $t = 0$ and $t = -t_0$ within the entire region in the form (2.1). For (3.4) yet another condition is necessary; therefore we put $w = 0$ for $t = -2t_0$, $z = 0$, $0 \leq r \leq 1$.

The scheme just presented was realized in the form of a program in the ALGOL-60 language. The calculations were carried out on a BÉSM-3M computer. The maximum number of points of the grid is 2500. Since at each point it was necessary to retain four quantities, the displacement blocks were not placed in the operational store, and they had to be kept on the drum. Exchange with the drum occupied roughly half the calculation time. The time necessary for the calculation of the solution for 0 to t was approximately $2 \cdot 10^{-4}\, z_0 R_1 t/h^3$ min.

4. The objective of the calculations was to establish the effect of the parameters of the problem on the deflection of the plate. As was already said at the beginning, while interpreting the results of measurement we assume that the deflection of the center of the plate is proportional to the stress in the medium. If we use σ to denote the stress being measured, then from the solution of the corresponding hydrostatic problem we find the expression for σ in terms of the displacement of the center of the plate:

$$\sigma = 64\, w\,(t, 0, 0)\,/\,I \tag{4.1}$$

If we do not take into account the effect of the deflection of the plate on the stress field, then in the reflected wave $\sigma_{zz} = 1$ for $t > 0$. Consequently, σ is equal to the ratio of the stress being measured to the true stress.

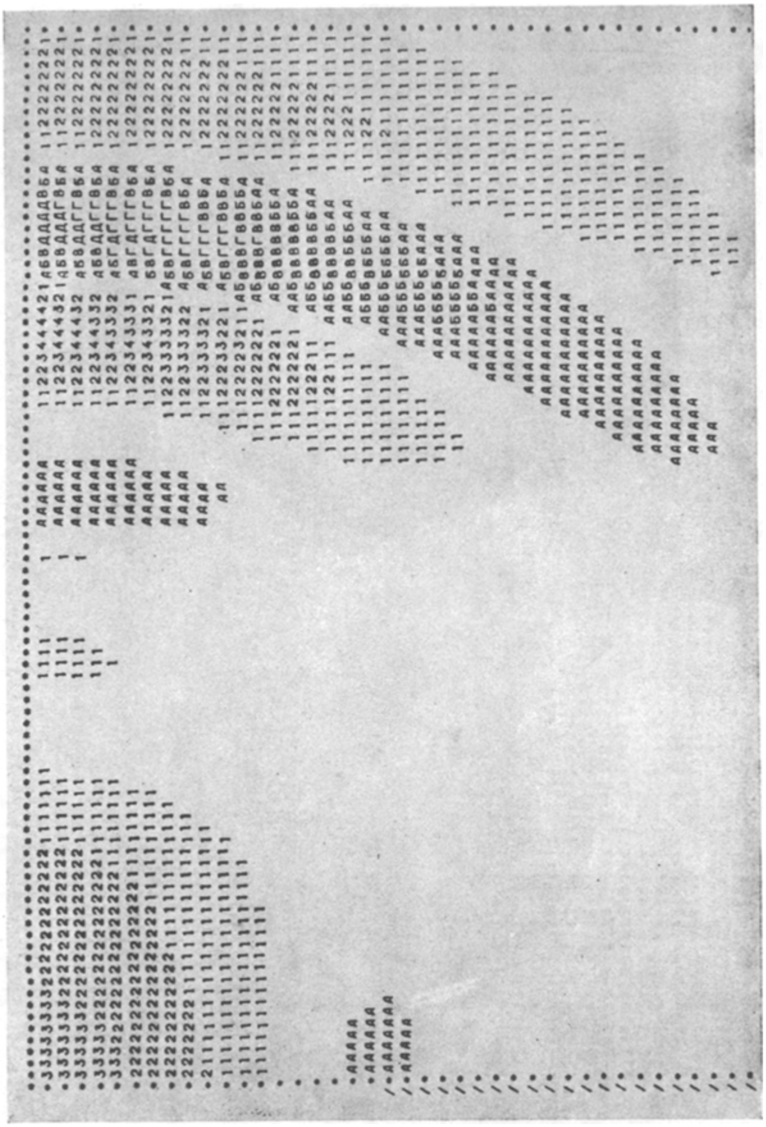

Fig. 8

Five physical parameters, μ, k, ρ, I, R, and three parameters characterizing the scheme selected, h, z_0, R_0, enter into the problem. The parameters of the scheme influence only the accuracy of the results, but the physical parameters influence σ in its character.

As it is not at all possible in a numerical calculation to establish the simultaneous effect of all parameters, we must investigate the effect of each parameter separately. In order not to write the same combinations of the parameters, in the sequel we put

$$\mu = 0.3,\ \ k = 0,\ \ \rho = 0.1,\ I = 0.02,\ \ R \to \infty,\ \ h = 0.1,\ \ z_0 = 3.0,$$
$$R_0 = 3.0$$

and talk only about the difference of the parameters from the values thus listed.

The effect of k is simplest of all to investigate, since it assumes only two values: 0 and 1. It was discovered that σ only slightly depends on k. In Fig. 2 the solid line depicts $\sigma(t)$ for $k = 0$; the points correspond to the solution with $k = 1$. Analogous results have been obtained for $I = 0.01$, 0.04, 0.08, and we can assume that k influences σ within the limits of a few per cent. This circumstance is considered to be important, since about real conditions of contact we usually know only that they lie somewhere between the conditions of sticking ($k = 1$) and total slipping ($k = 0$).

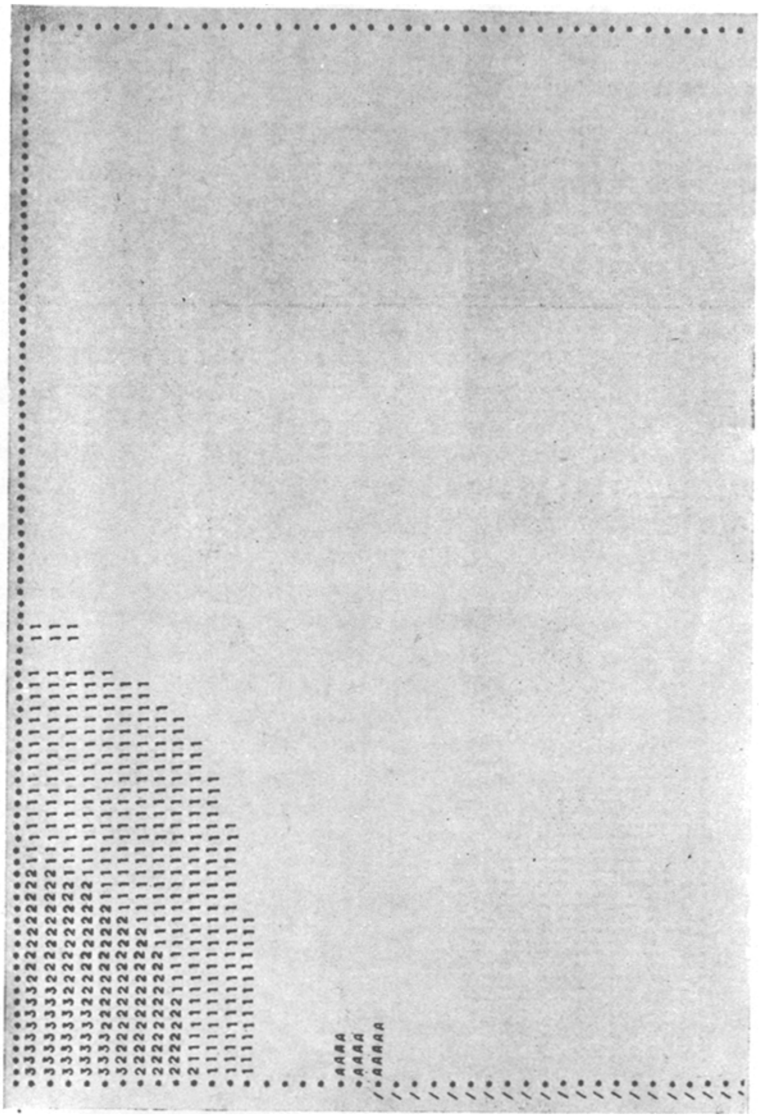

Fig. 9

The dimensionless density ρ in principle can be arbitrary, but for real transducers it is of the order 0.1. Since the solution of the static problem does not depend on ρ, the latter influences only the transient process. This effect is seen from Fig. 3, where $I = 0.01$, the curve 1 depicts $\sigma(t)$ for $\rho = 0.1$, the curve 2 depicts it for $\rho = 0.5$, and the points signify σ for $\rho = 0.02$. We see that for small ρ the quantity $\sigma(t)$ only slightly depends on ρ, while the duration of the transient process increases with increase in ρ, i.e., heavier plates have worse dynamic characteristics.

The effect of the side walls was also investigated. As was to be expected, it was maximum for small I, but even in this case it was always small. In Fig. 4, $I = 0.01$, the curve 1 represents $\sigma(t)$ for $R \to \infty$, the curve 2 represents it for $R = 1.1$, and the points correspond to $R = 3$.

The dependence of σ on μ and I is the greatest.

The effect of these parameters was investigated in [4], where a numerical solution of the static problem was set up for $R \to \infty$ and $k = 0$.

The same problem was solved by another method by E. B. Sretenskii,* who obtained the expression

$$\sigma_0 = [1 + 0.0431\mu\,(1 - \mu)/I]^{-1} \tag{4.2}$$

*E. B. Sretenskii, On the Theory of a Membrane Stress Transducer [in Russian], Diploma Thesis, Moscow Physicotechnical Institute (1970).

where the notation of the given work is used and σ_0 corresponds to $\sigma(\infty)$. It should be noted that the numerical results [4] are well described by the expression (4.2).

The limits of applicability of (4.2) were evaluated. For this the solution was calculated as far as $\sigma(t)$ did not become a constant. The constant σ_1 thus obtained was compared with σ_0 from (4.2). The results are presented below:

I	0.01	0.04	0.08	0.10	1.00	
σ_0	0.52	0.82	0.90	0.92	0.99	$\mu=0.3$
σ_1	0.53	0.83	0.91	0.93	1.01	

μ	0.00	0.10	0.30	0.50	
σ_0	1.00	0.84	0.69	0.65	$I=0.02$
σ_1	1.01	0.86	0.69	0.65	

Check calculations with increased accuracy showed that the difference between σ_0 and σ_1 is explained better by the errors of the numerical calculation than by the error of (4.2). Thus, we can assume that for $I \geq 0.01$ and any μ, (4.2) gives at least two correct digits after the decimal point.

From (4.2) we can see that the effect of μ is considerable for small I and that it falls as I increases. The situation is analogous in the dynamic case. In Fig. 5 we have depicted $\sigma(t)$ for $I=1$, $h=0.05$, $z_0=R_0=2.5$, where the curve 1 corresponds to $\mu=0.5$, $\rho=0.1$ and the curve 2 corresponds to $\mu=0.3$, $\rho=0.5$. The calculation for $\mu=0.01$ resulted in the coincidence of the curves 1 and 2 with an error less than 0.01. This means that for sufficiently stiff plates the motion is close to a one-dimensional motion.

The distribution of the stresses above the plate with time is illustrated by Figs. 6-9, where $\sigma_{zz}(r, z)$ is depicted at the time instants $t=1.0, 2.0, 3.0, 5.0$ for the following parameter combination: $\mu=0.3$, $I=0.1$, $\rho=0.1$. The quantity r is set off along the abscissa axis, while z is set off along the ordinate axis; the wall is marked by shading. The zones of reduced stresses are marked by the symbols 1, 2, 3, ...; the zones of increased stresses are marked by the symbols A, Б, В, Г. A transition to the next symbol corresponds to a variation of σ_{zz} by 0.05. In particular, the absence of a symbol signifies $\sigma_{zz}=1\pm0.025$; the symbol A signifies $\sigma_{zz}=1.05\pm0.025$, the symbol 1 signifies $\sigma_{zz}=0.95\pm0.025$, and so forth.

A check of the accuracy of the results was carried out by varying the parameters h, z_0, R_0. It was found that replacement of the infinite region by a finite one with the conditions (2.4) results in an insignificant contribution to the overall error. This error for the variants considered did not exceed 0.05.

Concluding, the author thanks the participants of the Seminar on Dynamics of Solids, at the Institute of Problems of Mechanics, for the discussion of the paper.

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